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Implementation of Vedic Algorithm for Solution of Simultaneous Equations on FPGA

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ABSTRACT

In our daily life, Mathematics plays very important role in science and technology as well as live research and predictions or forecasting is undividable part of our life. It is well known fact in conventional mathematics if, 'n' no of variable is to be determined we required '(n-1)' equations are required, in this regard Simultaneous Equations plays a major role. In other words they are the engines of sciences, various methods are used to solve simultaneous equations namely, Elimination, Substitution, Graphical, and Matrix. This paper is to study the Paravartya rule (cross-multiplication method) from Vedic mathematics, introduced by Jagadguru Swami Sri Bharati, Krishnan Tirthaji Maharaja [1] for solving systems of two linear equations with necessary modifications. The system has been tested on FPGA with emphasis on speed, area and power the results are tabulated and discussed.

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I. INTRODUCTION

In today's world, real time automation is a major goal, not a single application remains uncovered by electronics. VLSI and DSP are the two contemporary fields of Electronics, which plays crucial role in our most of the day to day applications. They consist of several blocks but in most of the cases multiplier block is a key component while deciding overall execution time of system. Since multiplication (complex) dominates the execution time of most of the algorithms, the need of high speed multiplier is obvious. Multiplication time is still the dominant factor in determining the instruction cycle time of many electronic chips. In DSP for time efficient mathematical operations uses convolution process. This gives a direct method of reducing manipulations processing time, high throughput and area efficient architecture on the FPGA with application of simultaneous equation.

The author(s) have worked out this paper in two sections, in the first section briefly about the type of system equations and in the second section a Vedic algorithm by which the solution has been determined. A system can be well defined by either linear or simultaneous set of equations which will enable the system study for its performance for the given set of inputs.

LINEAR EQUATION

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable. It can have one or more variables. They occur abundantly in most subareas of mathematics and especially in applied mathematics. While they arise quite naturally when modeling many phenomena, they are particularly useful since many non-linear equations may be reduced to linear equations by assuming that quantities of interest vary to only a small extent from some "background" state. Linear equations do not include exponents.

We will consider the case of a single equation for which one searches the real solutions. All its content applies for complex solutions and, more generally for linear equations with coefficients and solutions in any field.

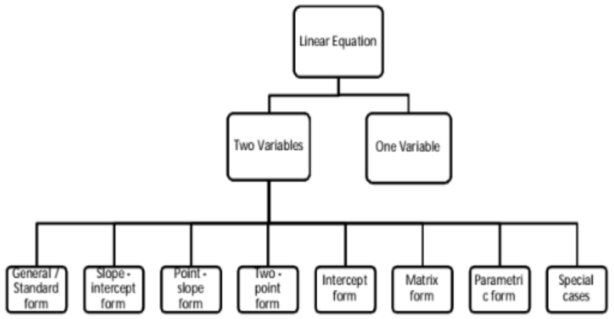

Fig 01. Different types of Linear Equations

II. SIMULTANEOUS EQUATIONS

Solving for two unknown variables along with two equations are generally known as simultaneous equations. The found values of these unknown variables satisfy both equations simultaneously. In general, if there will be X unknown variables, then X independent equations will require to obtain a value for each of those unknown variables.

A piece of mathematical equation with an equals sign in it, describes a graph. Various types of equations describe various graphs. For example, $y = mx + c$ (where m is the gradient of the line and c is the y-intercept) is the general straight line equation and $y = ax^2 + bx + c$ if for parabolas. Other functions have distinctive graphs. An engineer is expected to be familiar with the meaning of solving an equation or set of equations.

In order to solve a single equation one needs to have an equation and an unknown. Example, the equation $2y- 1 = 0$ as there is only one unknown y but you cannot solve an equation like $z = 2y - 1$ because there are two unknowns, z and y. To solve equations with two unknowns need two equations. Solution for a pair of equations such as:

$$z = 3y - 1 \quad \text{and} \quad z = 4y + 6$$

These both equations are in the form $y = mx + c$, so their graphs are straight lines, and there equations will be known as simultaneous equations. The solution to these types of simultaneous equations is usually a single value of z and a single value of y which represents a point of occurrence on graph. If point is on both the lines then the two lines must intersect at that point.

The solutions to simultaneous equations are/is intersecting points of these lines.

So the mathematics of simultaneous equations is the same as that of lines. In fact the Subject of solving simultaneous equations is called Linear Algebra – the algebra of lines.

We will have:

1) Find the point of intersection of two straight lines by solving simultaneous equations of the form $y = mx + c$.

2) Two parallel lines never intersect each other. As they are no intersecting points, system doesn't have any solution.

SOLVING LINEAR SYSTEM

It is extremely helpful to express linear simultaneous equations while solving in the form $y = mx + c$.

Just need to transpose equations to make y as the subject. The reason is, to identify the gradient and y-intercept of each graph easily. Then these values will be helpful to find the known. It will be also useful and convenient to sketch the lines for graphical representation of those lines.

A finite set of equations in the same unknowns of which the common solutions have to be determined.

1. Equations with (DGDI) different gradients and different y- intercepts
2. Equations with (DGII) different gradients and identical y- intercepts
3. Equations with (IGDI) identical gradients but different y- intercepts
4. Equations with (IGII) identical gradients and identical y- intercepts

There are several algorithms for solving a system of linear equations.

$$y = mx + c \text{ and } y = nx + d$$

Gradients	y-intercept	Type	Comments
$m \neq n$	$c \neq d$	1	The lines cross at a single point not
$m \neq n$	$c = d$	2	The lines cross at the y-intercept (0, c) = (0, d)
$m = n$	$c \neq d$	3	The lines are parallel and so you get no solutions
$m = n$	$c = d$	4	The lines are identical and so you get infinite solutions

Also there exist several ways to solve the simultaneous equations, those are

1. Substitution method,
2. Elimination of variables,
3. Graphical,
4. Cramer's rule,
5. Vedic Math's Method.

III. VEDIC MATHEMATICS

The Vedic mathematics is based on 'Veda' which is nothing but 'to know without limit'. The word Veda covers all known- unknown to humanity. The Veda is a Library of all knowledge. Vedic mathematics is part of four Vedas (books of wisdom). It is based on Sthapatya- Veda (book on civil engineering and architecture), which is an upa-veda (supplement) of Atharva Veda. It covers maximum mathematical operations like arithmetic, geometry, trigonometry, quadratic equations, factorization and even calculus. Vedic mathematics based all mathematical operations are summarized and explained around 16 sutras (formulae) and 16 Upa-sutras (sub formulae) from Atharva Veda. Vedic mathematics is optimized solutions to mathematical operations with having its mathematical analysis as proof. That's why VM has such a degree of eminence which cannot be disapproved.

From Vedic mathematics' there are 4 Sutras and one sub-sutra are used for calculation of Linear equations which improves speed of operation and those are,

1. Paravartya Yojayet: Transpose and adjust (or apply)
2. Anurupye sunyam anyat: If one is in ration, the other one is zero
3. Sankalana: Vyavakalanabhyam: By addition and by substraction
4. Sopantya: dvyam antyam: The ultimate and twice the penultimate
5. Antyayor eva: Only the last term

IV. METHODOLOGY USED IN PROPOSED SYSTEM

The 'Paravartya' is made up with 'para and vartya'. Generally 'para' is prefix word had its meaning is 'aside / away'. 'Vrt' is root for 'vartya' and has meaning 'can be exchanged'. So collectively it means 'having transposed'. 'Yojayet' has different meanings in different fields. We will look it from mathematics way and it means 'join to / connect to / to adjust'. Therefor collectively meaning of first sutra is 'Transpose and adjust / apply'.

'Paravartya – Yojayet' also means 'transpose and apply or adjust'.

ALGEBRIC PROOF OF CONCEPT

General representation of linear simultaneous equation are, $cx1M + cy1N + L1 = 0$ ……. (Equation A)

$cx2M + cy2N + L2 = 0$ ……. (Equation B)

To find the values of unknown M, we will multiply Equation A by 'cy2' and Equation B by 'cy1', hence new equations will be,

$(cy2 * cx1) M + (cy2 * cy1) N + (cy2 * L1) = 0$

…… (Equation A_1)

$(cy1 * cx2) M + (cy1 * cy2) N + (cy1 * L2) = 0$

…… (Equation B_1)

Now subtracting (Equation B_1) from (Equation A_1), we will get as,

$(cy2 * cx1) M + (cy2 * cy1) N + (cy2 * L1) = 0$

…… (Equation A_1)

$(cy1 * cx2) M + (cy1 * cy2) N + (cy1 * L2) = 0$

…… (Equation B_1)

$[(cy2 * cx1) - (cy1 * cx2)] M + (cy2 * L1) - (cy1 * L2) = 0$

Separating unknown and constants, we found,

$$M = \frac{(cy1 * L2) - (cy2 * L1)}{(cy2 * cx1) - (cy1 * cx2)}$$

…… (Equation A_3)

To find the values of unknown N, we will multiply Equation A by 'cx2' and Equation B by 'cx1', hence new equations will be,

$(cx2 * cx1) M + (cx2 * cy1) N + (cx2 * L1) = 0$

…… (Equation A_2)

$(cx1 * cx2) M + (cx1 * cy2) N + (cx1 * L2) = 0$

…… (Equation B_2)

To find the values of unknown N, we will multiply Equation A by 'cx2' and Equation B by 'cx1', hence new equations will be,

$(cx2 * cx1) M + (cx2 * cy1) N + (cx2 * L1) = 0$

…… (Equation A_2)

$(cx1 * cx2) M + (cx1 * cy2) N + (cx1 * L2) = 0$

…… (Equation B_2)

Now subtracting (Equation B_2) from (Equation A_2), we will get as,

$(cx2 * cx1) M + (cx2 * cy1) N + (cx2 * L1) = 0$

…… (Equation A_2)

$(cx1 * cx2) M + (cx1 * cy2) N + (cx1 * L2) = 0$

…… (Equation B_2)

$[(cx2 * cy1) - (cx1 * cy2)] N + (cx2 * L1) - (cx1 * L2) = 0$

Further solving this equation, we will get,

$[(cx2 * cy1) - (cx1 * cy2)] N = (cx2 * L1) - (cx1 * L2)$

Separating unknown and constants, we found,

$$N = \frac{(cx2 * L1) - (cx1 * L2)}{(cy2 * cx1) - (cy1 * cx2)}$$

…… (Equation B_3)

Condition C1: Denominator not equal to zero.

That is $(cy2 * cx1) - (cy1 * cx2) \neq 0$

$(cy2 * cx1) \neq (cy1 * cx2)$

$\boldsymbol{case\ 01}$: $For\ \frac{cx1}{cx2} \neq \frac{cy1}{cy2}$

This pair of linear simultaneous equations has unique pair of solution. This is known as consistent equations. So the solutions for above condition can be represented by (Equation A_3) and (Equation B_3).

$$M = \frac{(cy1 * L2) - (cy2 * L1)}{(cy2 * cx1) - (cy1 * cx2)}$$

$$N = \frac{(cx2 * L1) - (cx1 * L2)}{(cy2 * cx1) - (cy1 * cx2)}$$

For simplicity this can be represented as,

$$\frac{M}{(cy1 * L2) - (cy2 * L1)} = \frac{N}{(cx2 * L1) - (cx1 * L2)} = \frac{1}{(cy2 * cx1) - (cy1 * cx2)}$$

$\boldsymbol{Case\ 02}$: $\quad For\ \frac{cx1}{cx2} = \frac{cy1}{cy2} \neq \frac{L1}{L2}$

The system has no solutions at all and the lines drawn with these coefficients are always parallel to each other.

$\boldsymbol{Case\ 03}$: $\quad For\ \frac{cx1}{cx2} = \frac{cy1}{cy2} = \frac{L1}{L2}$

The system has infinite solutions and lines drawn using above coefficients are always coincident with each other.

Condition C2: Denominator is equal to zero.

That is $(cy2 * cx1) - (cy1 * cx2) = 0$

$(cy2 * cx1) = (cy1 * cx2)$

$$\frac{cx1}{cx2} = \frac{cy1}{cy2}$$

$$\frac{cx1}{cx2} = \frac{cy1}{cy2} = ck \ \ldots where\ ck \neq 0$$

$\frac{cx1}{cx2} = ck$; $\quad cx1 = ck * cx2 \quad$ and

$\frac{cy1}{cy2} = ck$; $\quad cy1 = ck * cy2$

$\boldsymbol{Case\ 01}$: $\quad For\ L1 = L2 * ck$

Putting this value in

(ck * cx2) M + (ck * cy2) N + (L2 * ck) = 0

$ck\ (cx2 * M + cy2 * N + L2) = 0 \qquad where\ (k \neq 0)$

$(cx2 * M + cy2 * N + L2) = 0$

which is clearly equation B_1

The solution coefficients will be $\frac{cx1}{cx2} = \frac{cy1}{cy2} = \frac{L1}{L2}$

so, lines are coincident and system has infininte solutions.

$\boldsymbol{Case\ 02}$: $\quad For\ L1 \neq L2 * ck$

Putting this value in (ck * cx2) M + (ck * cy2) N + L1 = 0

$ck\ (cx2 * M + cy2 * N)\ + L1 = 0$

$ck\ (-L2) + L1 = 0$ from equation B_1

$L1 \neq L2 * ck$ which is not true as per our assumption in above case 02.

The solution coefficients will be $\frac{cx1}{cx2} = \frac{cy1}{cy2} \neq \frac{L1}{L2}$

so, lines are parallel and system has no solutions.

V. RESULTS AND DISCUSSIONS

The simulation results shown below are the outputs for the 2 variable linear simultaneous equation simulated in Xilinx ISE tool for the device XC3S400PQ208 Sparten-III FPGA. Below shown 'Device utilization summary' it-self speaks about the efficient use of Vedic Mathematics while finding the unknown for the linear simultaneous equations.

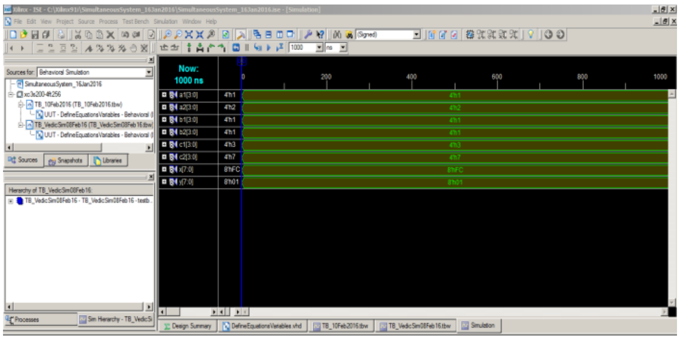

Figure 02: Simulation result for linear simultaneous equation

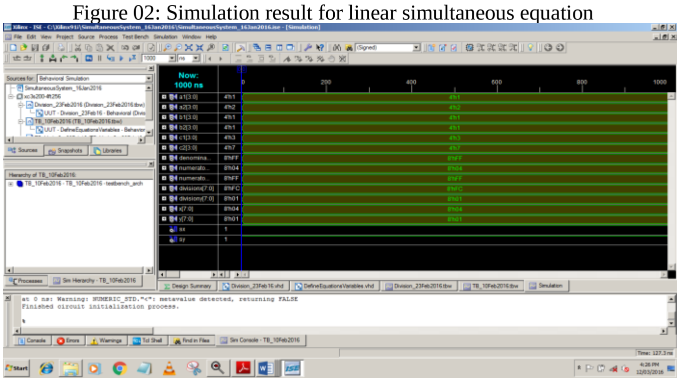

Figure 03: Simulation result for linear simultaneous equation with intermediate results.

VI. CONCLUSION

From above results we may conclude that, finding unknowns for the 2 variable linear simultaneous equations can be mechanized and embedded in the electronic logic circuitry and also can be formulated in SoC (System on Chip) with will maximize the efficiency and throughput. As we increase the number of unknowns then the results with this method are more efficient and consistent in terms of delay, than any other existing methods. More over there is hardly any method exists for more than two variables.

Device utilization summary:

Selected Device: XC3S400PQ208.

Number of Slices:	36	out of	1920	1%
Number of 4 input LUTs:	68	out of	3840	1%
Number of IOs:	82			
Number of bonded IOBs:	82	out of	173	47%
Number of MULT18X18s:	6	out of	12	50%

REFERENCES

[1] Krishnan Tirathji Maharaja Jagadguru Swami Sri Bharati, Vedic Mathematics, MotilalBanarsidass Publishers, Delhi, revised Ed (1992), 2001.

[2] D.K.R. Babajee "Solving Systems of linear equations using the Paravartya rule in Vedic Mathematics", Allied Network for Policy Research & Advocacy for Sustainability, Mauritius IEEE Subsection, Mauritius.

[3] S. Das, "Solution of Linear Algebra Equations and sums of fractions- additions using SUTRA method", Indian Journal of History of Science 29(4), 1994, BARC Bombay.

[4] Dani, S.G. Vedic Mathematics; Facts and Myths, Frontline, 22nd Oct. 1993.

[5] N Jamil. A Comparison of direct and indirect solvers for linear systems of equations. Int. J. Emerg. Si, 2(2):310 321, 2012.

[6] http://www.vedicmaths.org

